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| **MATHEMATICS DEPARTMENT**  **Year 11 Specialist – 2016**  **Test Number 2: Mathematical Proof**  **Resource Free** |

**Name: \_\_\_\_\_\_\_\_\_\_SOLUTIONS\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ Teacher: DDA**

**Marks: 39**

**Time Allowed: 40 minutes**

**Weight: 2.5%**

**Instructions:** You **ARE** **NOT** permitted any notes or calculator. Show your working where appropriate remembering you must show working for questions worth more than 2 marks.

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1 Consider the statements:

I   If you have a driver’s licence then you are over 16.

II If you are not over 16 then you do not have a driver’s licence.

Which of the following is true?

A  II is the converse of I.

B II is the contrapositive of I.

C II is a counterexample of I.

D II is the negative of I.

E  II is the reverse of I.

[1 mark]

2 Consider the argument:

All fish live in water.

A turtle lives in water.

Therefore a turtle is a fish.

The argument is:

A   a contrapositive

B  inductive

C a contradiction

D  invalid

E a counterexample.

[1 mark]

3 If  Description: symb3  is true then which statement is true?

A  P Description: symb3 Q

B P Description: symb2 Q

C Q Description: symb3 P

D Q Description: symb2 P

E   Description: symb3 

[1 mark]

4 The values x = 3, y = −2 would be a counterexample to which of the following?

A x > y Description: symb3 x2 > y2

B x > y Description: symb3 

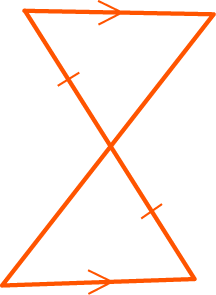
C x > y Description: symb3 xy > y2

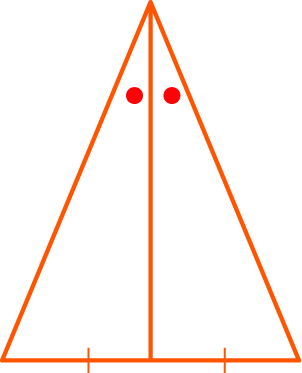
D x > y Description: symb3 x2 > xy

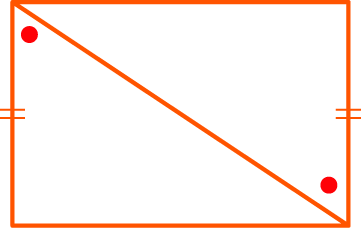
E  x > y Description: symb3 

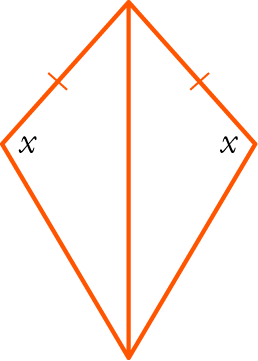
[1 mark]

5 You would use the SAS test to prove which pair of triangles congruent? (Circle the correct answer)

A 

B 

C 

D 

E   None of the above.

[1 mark]

6 Write the statement below using mathematical notation.

For all rational numbers x there exist integers p and q where q is non-zero such that .

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| ∀x ∈ Q, [1 mark] ∃ p, q ∈ Z, q ≠ 0 [1 mark] : |
|  |

[3 marks]

7 For the statement:

If you drink too much alcohol then you will get alcohol poisoning.

a  Write the converse.

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| If you get alcohol poisoning then you drink too much alcohol. |
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b   Write the inverse.

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| If you do not drink too much alcohol then you will not get alcohol poisoning. |
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c  Is the statement an equivalence? Give reasons.

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| Statement is true and converse is true [1 mark], therefore statement is an equivalence. |
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[4 marks]

8 Consider the two premises:

P: A quadrilateral is a rhombus.

Q: A quadrilateral has diagonals that bisect each other at right angles.

a Write P Description: symb3 Q.

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| If a quadrilateral is a rhombus then the quadrilateral has diagonals that bisect each other at right angles. |

b Write Q Description: symb3 P.

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| If a quadrilateral has diagonals that bisect each other at right angles then the quadrilateral is a rhombus. |

c Is it true that P Description: symb2 Q? If so, write a statement linking P and Q using iff.

True or False? True (no mark)

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| A quadrilateral has diagonals that bisect each other if and only if the quadrilateral is a rhombus. |
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[3 marks]

9 Complete the proof that  is irrational.

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| Proof: Assume that  is rational, i.e. ∃ m, n ∈ Z, n ≠ 0 such that, |
| where m, n have no common factors. |
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| ∴ is irrational. |

[6 marks]

10 For each statement below, consider the contrapositive statement to determine whether or not the statement is true. Justify your reasoning.

a If pensioners have electronic devices then they send emails.

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| Contrapositive: If pensioners do not send emails then they do not have electronic devices. |
| Contrapositive may not be true. They may have electronic devices that do not do emails or use the library computer to send emails. |
| If the contrapositive is not true the original is not true. |

![](data:None;base64,)

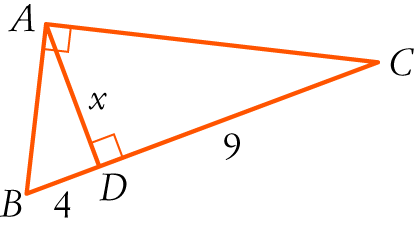
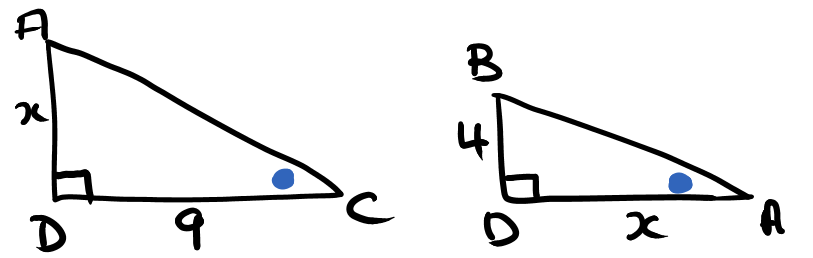
![](data:None;base64,)b If a number is divisible by 4 then it is even.

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| Contrapositive: If a number is not even then it is not divisible by 4. |
| Contrapositive is true. |
| If the contrapositive is true the original is also true. |

![](data:None;base64,)

[5 marks]

11 In the diagram, AB ⊥ AC, AD ⊥ BC, BD = 4, DC = 9, AD = x.



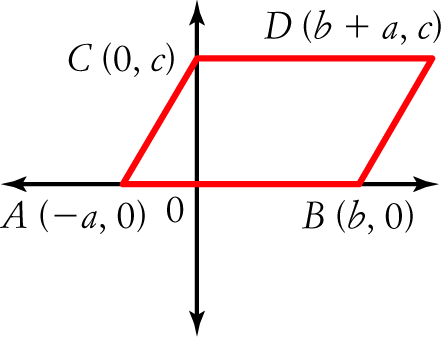
a Prove that Δ ABD ||| Δ CAD. Make sure you justify every line with a reason or rule.

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| In and : |
| ( both ) |
| ( both complementary to ) |
| ( AA ) |

b Hence find the value of x.

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| (matching sides in similar triangles are in proportion) |
| ∴ x2 = 36 |
| ∴ x = 6 (x > 0) |

[6 marks]





**12** Consider the diagram with coordinates (−a, 0), (b, 0), (0, c), (b + a**,** c) representing the points A, B, C, D respectively, such that ABDC is a parallelogram.

a Determine the length of each of the following in terms of and .

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| *AB =* | *CD =* |
| *AC =* | *BD =* |
| *AD =* | *BC =* |

**b** Hence, write an expanded expression for each of the following in terms of and

**i)** AB2 + DB2 + DC2 + CA2

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ii) AD2 + CB2

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c What property of a parallelogram does this demonstrate?

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| The sum of the squares of the diagonals equals the sum of the squares of the side lengths. |
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[2,2,2,1: 7 marks]